FRICTION LOSSES IN THE MOTION OF AIR-WATER AND AIR-SOLUTION MIXTURES IN CIRCULAR VERTICAL TUBES AT LOW PRESSURE

I. I. Sagan, N. Yu. Tobilevich, and S. I. Tkachenko

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Results of experimental investigations of the friction losses in the motion of air-water and viscous air-solution mixtures in vertical circular tubes at low pressure are presented. Formulas are given for calculating friction losses in the range of test parameters investigated.

The analytical and experimental study of the hydrodynamics of two-phase mixtures is a very complex problem. At low pressure, when the process takes on a fluctuating character, the problem is even more complicated. So far, the chief method of study of two-phase media has been experiment and its theoretical basis, similarity theory.

Attempts have been made, however, to subject certain types of two-phase flow to an orderly analytical study. Armand and Kutateladze independently performed studies of the flow of a mixture with a liquid film. Shvab and Konstantinov [1] have examined the case of slug flow.

We shall dwell in more detail on the hydrodynamic picture of two-phase flow in a circular verticle tube, as related to the operating conditions of evaporators and vaporizers in the sugar industry. At low pressure small reduced and relative gas velocities (1–1.5 m/sec) correspond to slug type motion in two-phase flow; core-type motion is observed only at considerable reduced gas velocities, on the order of 10–15 m/sec.

According to our calculations, the evaporaters used in the sugar industry operate at reduced vapor velocities in the range 0–30 m/sec in evaporating low- and high-viscosity solutions. In these conditions it is possible to have, in a single section of tube, the alternate transient existence of both types of motion. In an experimental equipment, in the transition region between the two types of motion, we visually observed a large amount of wave formation and unsteadiness of the phase separation boundary; masses of liquid, separating from the walls, partition off the flow, therefore the section along which the gas can pass is reduced, the dynamic head of the gas increases, and the gas, striking the liquid masses, forces them against the wall, while simultaneously changing its direction of motion, i.e., considerable mass transfer in the transverse direction is observed. The increase in reduced gas velocity creates a film of liquid component at the wall, but even at considerable velocities some wave formation is noticeable at the surface of the liquid. Investigations carried out with air-water flows in horizontal and vertical tubes [2] showed that the resistance law depends on the position of the tube.

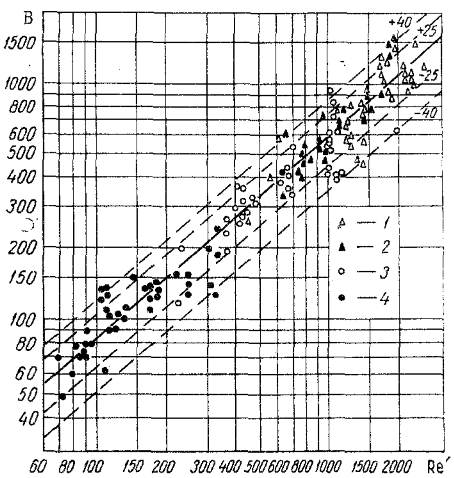

Fig. 1. Correlation of the author's test data on friction losses of air-solution flows in a vertical circular tube with d = 0.0327 m, for a sugar solution $\left(B \equiv \frac{\Delta P_f / \Delta P_0}{(W_0''/W_0')(\gamma''/\gamma')(Fr_m/[Fr_m]_a)^{-0.22}} = f(Re') \right)$: 1) with DS = 51.5%, P = $10.5 \cdot 10^4$–$12.7 \cdot 10^4$ N/m^2; 2) respectively, 51.5% and $5.5 \cdot 10^4$–$8.05 \cdot 10^4$ N/m^2; 3) 62% and $10.7 \cdot 10^4$–$12.7 \cdot 10^4$ N/m^2; 4) 68.7% and $11.4 \cdot 10^4$–$13.1 \cdot 10^4$ N/m^2.

Table 1
Characteristics of the Media Investigated

Liquid	Gas	t, °C	$\gamma' \cdot 10^{-3}$, N/m^2	$\nu' \cdot 10^6$ m^2/sec	$\sigma' \cdot 10^2$ N/m
water	Air	30	9.76	0.71	7.11
sugar solution DS = 51.5%	"	30	12.1	9.25	7.41
sugar solution DS = 62%	"	35	12.6	28.9	7.54
sugar solution DS = 68.7%	"	30	13.1	91.5	7.74

In the above-mentioned cases the gravity forces have different effects; in vertical tubes, in some cases, they cause reverse flow of liquid, and in horizontal tubes—flow asymmetry. The friction losses of two-phase flows with a highly viscous liquid component have been determined only in a horizontal tube [1].

With the object of studying friction losses in two-phase flow in vertical tubes, we built two experimental rigs which differed in the diameters of the experimental and stabilizing sections. The rigs formed a vertical circuit with natural circulation, and it was possible to study the process during forced motion of the liquid component. There were three pressure

Table 2
Characteristics of the Tests

Liquid	$p\cdot10^{-4}$, N/m^2	$(Re')_{min}$	$(Re')_{max}$	$\frac{\gamma''}{\gamma'}\cdot10^3$	$\left(\frac{W_0''}{W_0'}\right)_{min}$	$\left(\frac{W_0''}{W_0'}\right)_{max}$	$(Fr_m)_{min}$	$(Fr_m)_{max}$	d, m
water	>9.81	4650	19100	1.27 —1.56	1.25	22.5	33	730	0.0122
water	>9.81	15000	57500	1.22 —1.42	1.5	30.0	12	1470	0.0327
water	<9.81	15000	57500	0.738—1.05	1.6	22.5	25	265	0.0327
sugar solution DS = 51.5%	>9.81	460	4410	1.01 —1.23	1.2	217	7.4	2760	0.0327
sugar solution DS = 62%	>9.81	225	1360	0.99 —1.18	1.25	137	21	2015	0.0327
sugar solution DS = 68.7%	>9.81	70	330	1.01 —1.17	1.63	132	12.5	1300	0.0327
sugar solution DS = 51.5%	<9.81	630	4410	0.53 —0.78	2	66.6	42.6	344	0.0327

taps in the experimental section, and special fast-close valves were installed at its beginning and end. The valves did not cause any flow disturbance and cut off the flow in 0.015 sec. The length of the stabilizing section was l = 50d.

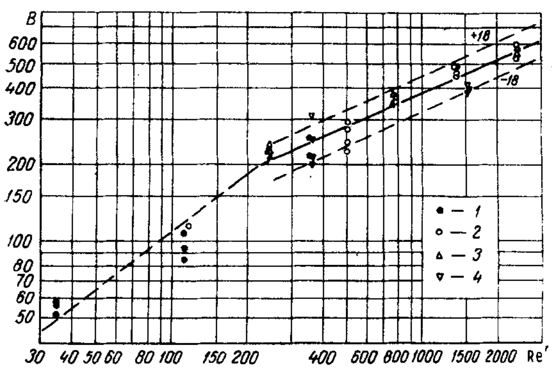

Fig. 2. Dependence of $B = \frac{\Delta P_f}{\Delta P_0} \Big/ \frac{W_0''}{W_0'}\,\frac{\gamma''}{\gamma'} = f(\mathrm{Re}')$ for horizontal tubes with P = 9.81 · 10^4 N/m^2, d = = 0.0327 m: 1) for Fr_m = 0.04; 2—0.33; 3—1.4; 4—4.

Method of determining friction losses in vertical tubes. Tests were done in vertical tubes with d = = 0.0327 and 0.0122 m, in which air-water and air-solution mixtures were in motion, with pressures in the middle of the experimental section of P = 5.5 × × 10^4–13.1 × 10^4 N/m^2. The tube diameters were determined by a volume method, and the roughness was checked at the start and the end of the tests. In performing the tests, the total drop ΔP along the tube was measured with a piezometer or tubular differential manometers, and the specific weight of the mixture γ_m by the cutoff method. The friction losses were determined from the equation

$$\Delta P_f = \Delta P - \Delta P_{acc} - \Delta P_w. \qquad (1)$$

In reducing the test data obtained in the larger tube (d = 0.0327 m) the specific weight was taken not according to the instantaneous value of the true gas content φ_m at the instant of cutoff, but according to the mean; the losses in acceleration were calculated from the formula

$$\Delta P_{acc} = \frac{\gamma W_0^2}{g}\;\frac{\varphi_2 - \varphi_1}{(1-\varphi_1)(1-\varphi_2)}. \qquad (2)$$

The friction losses in the smaller tube (d = 0.0122 m), for the same values of the mass flow parameters, were considerably greater than in the larger (d = = 0.0327 m) tube, and therefore we though it possible to determine γ_m from the instantaneous value of φ_m. The calculated value of the losses in acceleration cannot introduce serious errors, since ΔP_{acc} did not exceed 2–3% of ΔP_f and ΔP_w in the conditions under which the tests were made.

The data of the test are presented in Tables 1 and 2.

Discussion of the test results. Exact analytical solutions, which have been developed only for certain regimes of two-phase flow [1], are extremely schematic, do not solve engineering problems, and do not permit evaluation of the degree of influence of the basic parameters on the flow friction and relative phase velocity.

By analysis of our test data, we managed to elucidate the influence of the individual parameters on the process, and to divide it into three regions distinguished by the nature of the influence of these parameters. The boundaries of the regions are determined by values of Re'. For the first region Re' = 70–2500, for the second 2500–5000, and for the third Re' > > 5000.

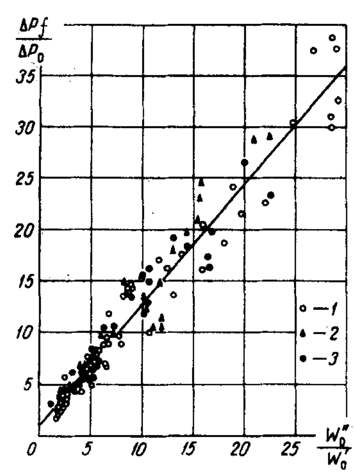

Fig. 3. Dependence of $\Delta P_f/\Delta P_0 = f(W_0''/W_0')$ for an air-water mixture: 1) with d = 0.0327 m, P = 10.2·10^4–11.9·10^4 N/m^2; 2) 0.0327 m and 6.16·10^4– 8.7·10^4 N/m^2; 3–0.0122 and 13·10^4–10.6·10^4.

Let us examine the first region. Solving the problem of friction losses of two-phase flow with a laminar liquid layer in horizontal tubes, Kutateladze [1] arrived at a specific combination of parameters W_0''/W_0', Re' and γ''/γ'. He noted that the Fr number of the liquid component had a small influence on the process when $\frac{W_0''}{W_0'}\sqrt{\mathrm{Re}'\frac{\gamma''}{\gamma'}} < 40$, and for the case

$$\frac{W_0''}{W_0'}\sqrt{\mathrm{Re}'\frac{\gamma''}{\gamma'}} > 40$$

the calculation formula

$$\frac{\Delta P_f}{\Delta P_0} = 1 + 0.1\left(\frac{W_0''}{W_0'}\sqrt{\mathrm{Re}'\frac{\gamma''}{\gamma'}}\right)^{1.3} \qquad (3)$$

was recommended.

In our tests in a vertical tube with d = 0.0327 m at Re' ≤ 2500, the Fr number was not varied over such wide limits as for the horizontal tubes [1], but even these limits in the horizontal tube conditions gave a variation of the ratio $\Delta P_f/\Delta P_0$ of about a factor of two.

Analysis of the test data and visual observations show that the flow structure in vertical tubes is largely determined by the parameter $\mathrm{Fr_m}$ when its value is less than 300–350. Increase of $\mathrm{Fr_m}$ leads to stabilization of the layer near the wall, to decreased mass transfer in the transverse direction, and to change in the ratio of descending and ascending liquid. When $\mathrm{Fr_m} > 300$–350 the process goes on in a more or less stable manner. Change of the above parameter slightly changes the hydrodynamic picture in the vertical tube, and the friction losses are observed to be independent of $\mathrm{Fr_m}$.

In horizontal tubes the boundaries of the self-similar region were determined at the same limits.

Variation of the $\mathrm{Fr_m}$ parameter for large specific weight of gaseous component more accurately characterizes a change in the ratio of inertia and gravity forces, than for the case of small specific weight of gas. The above ratio is determined more practically by a combination of $\mathrm{Fr_m}$ and γ''/γ'. To determine the friction losses of two-phase flows in vertical tubes at Re' ≤ 2500, we assumed a combination of the parameters W_0''/W_0', Re', $\mathrm{Fr_m}$, γ''/γ'.

It turned out that the influence of $\mathrm{Fr_m}$ may conveniently be expressed by the ratio $\mathrm{Fr_m}/[\mathrm{Fr_m}]_a$, where $[\mathrm{Fr_m}]_a$ satisfies the values of parameter $\mathrm{Fr_m}$, above which the self-similar region with respect to $\mathrm{Fr_m}$ begins.

Our experimental points are plotted in Fig. 1 in the logarithmic coordinates

$$\frac{\Delta P_f}{\Delta P_0} \Big/ \frac{W_0''}{W_0'}\frac{\gamma''}{\gamma'} \times \left(\frac{\mathrm{Fr_m}}{[\mathrm{Fr_m}]_a}\right)^{-0.22} = f(\mathrm{Re}').$$

In the region Re' = 70–2500 the test points are approximated by the equation

$$\frac{\Delta P_f}{\Delta P_0} = 1.62\frac{W_0''}{W_0'}\frac{\gamma''}{\gamma'}(\mathrm{Re}')^{0.86} \times \left(\frac{\mathrm{Fr_m}}{[\mathrm{Fr_m}]_a}\right)^{-0.22}. \qquad (4)$$

The main mass of points gives a scatter in the limits ±25%, and in the friction losses $\Delta P_f \le 1500$ N/m² the scatter was ±40%. This is explained by increase of the relative error with the same value of absolute error; moreover, for small drops even the absolute error of the measurements increases due to increase in fluctuations.

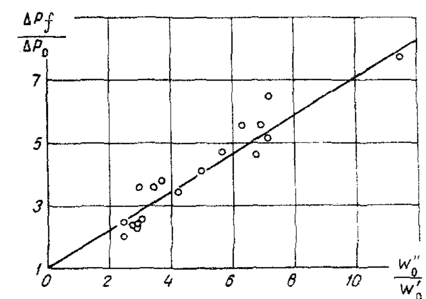

Fig. 4. Dependences of $\Delta P_f/\Delta P_0 = f(W_0''/W_0')$ for a sugar solution (DS = 51.5%)—Air with d = 0.0327 m, Re' = 3100–3960, P = 7.58 · 10⁴–12.4 · 10⁴ N/m².

For $\mathrm{Fr_m} < [\mathrm{Fr_m}]_a$ we use formula (4) in its original form, and when $\mathrm{Fr_m} > [\mathrm{Fr_m}]_a$ the ratio $(\mathrm{Fr_m}/[\mathrm{Fr_m}]_a)^{-0.22}$ drops out and the formula takes the form

$$\frac{\Delta P_f}{\Delta P_0} = 1.62\frac{W_0''}{W_0'}\frac{\gamma''}{\gamma'}(\mathrm{Re}')^{0.86}, \qquad (5)$$

i.e., for $\mathrm{Fr_m} < [\mathrm{Fr_m}]_a$, we have in mind that the ratio

$$\frac{\Delta P_f}{\Delta P_0} \Big/ \frac{W_0''}{W_0'}\frac{\gamma''}{\gamma'}\left(\frac{\mathrm{Fr_m}}{[\mathrm{Fr_m}]_a}\right)^{-0.22},$$

has been plotted on the axis of ordinates, and for $\mathrm{Fr_m} > [\mathrm{Fr_m}]_a$

$$\frac{\Delta P_f}{\Delta P_0} \Big/ \frac{W_0''}{W_0'}\frac{\gamma''}{\gamma'}.$$

It may be seen from the structure of (4) that it does not satisfy conditions at the boundary when $W_0'' = 0$, i.e., during single-phase flow.

Use of (4) is recommended in the range of variation of the dimensionless groups shown in Table 2. For possible comparison of the friction losses in vertical and horizontal tubes, we recomputed the friction loss data in two-phase flow in horizontal tubes from the coordinates

$$\xi_0'\,\mathrm{Re}' = f\left(\frac{W_0''}{W_0'}\sqrt{\mathrm{Re}'\frac{\gamma''}{\gamma'}\cdot 25}\right)$$

into the coordinates

$$\frac{\Delta P_f}{\Delta P_0} \Big/ \frac{W_0''}{W_0'} \frac{\gamma''}{\gamma'} = f(\mathrm{Re}')$$

for the case when the majority of points satisfies the condition

$$\frac{W_0''}{W_0'} \sqrt{\mathrm{Re}' \frac{\gamma''}{\gamma'}} < 40; \ \mathrm{Fr} = 0.04 \div 4; \ \mathrm{Fr_m} \geqslant [\mathrm{Fr_m}]_a,$$

where $[\mathrm{Fr_m}]_a$ was assumed to be 313 according to our test data. The correlation indicated above

$$\xi_0' \mathrm{Re}' = f\left(\frac{W_0''}{W_0'} \sqrt{\mathrm{Re}' \frac{\gamma''}{\gamma'}} \cdot 25\right)$$

was presented in [1] and is a generalization of the test data of Martinelli, Boulter, Taylor, and Morrison. In recomputing the physical properties of the components, γ'', ν', γ' were determined, and mass flow parameters were assigned for the case of mixtures of sugar solutions of concentrations of 51.2, 62, and 68.7% with air.

For a horizontal tube in the range of variation of Re' from 35 to 2500, two regions may be discerned. For Re' = 240–2500 the relation

$$\frac{\Delta P_f}{\Delta P_0} = \frac{W_0''}{W_0'} \frac{\gamma''}{\gamma'} (\mathrm{Re}')^{0.45}$$

is valid.

The majority of the points of Fig. 2 gives a scatter in the limits ±18%. The friction losses in vertical and horizontal tubes with Re' = 240–400 do not differ appreciably; as Re' increases the difference increases, and when Re' = 2000 the friction losses in a vertical tube are twice as great as in a horizontal one.

In the second region (Re' = 35–240) the nature of the dependence changes. The ratio $\Delta P_f/\Delta P_0$ in the vertical tube becomes somewhat less than in the horizontal one. This may be explained by the method of determining ΔP_0. The resistance coefficient for a laminar regime is assumed to be given by the formula $\lambda = 64/\mathrm{Re}$. Investigations carried out in the intake stream flow in circular tubes showed that at low Re numbers (Re = 0.5–550) the resistance law for single-phase flow varies, and the coefficient is given with sufficient accuracy the the relation [3]

$$\lambda = 132/\mathrm{Re}^{1.12}. \tag{6}$$

When Re < 250 the relation $\lambda = 64/\mathrm{Re}$ gives considerably underestimated values of λ.

For a vertical tube with a sugar solution-air mixture (DS = 68.7%) we calculated the resistance coefficient in single-phase flow from the formula $\lambda = 132/\mathrm{Re}^{1.12}$. When the formula $\lambda = 64/\mathrm{Re}$ was used, the ratios $\Delta P_f/\Delta P_0$ with Re' < 250 did not differ appreciably for the vertical and horizontal tubes.

When Re' ≥ 5000, the influence of the parameters Re' and $\mathrm{Fr_m}$ on the quantity $\Delta P_f/\Delta P_0$ is not very appreciable. The test data obtained in the vertical tubes with d = 0.0327 m (Re' ≥ 15000) and d = 0.0122 m (Re ≥ 4650) for the case of motion of air-water mixtures at $P = 6.17 \cdot 10^4 – 13.1 \cdot 10^4$ N/m² are approximated by the equation (Fig. 3)

$$\Delta P_f/\Delta P_0 = 1 + 1.18\ W_0''/W_0'. \tag{7}$$

Expressing these results in coordinates $\Delta P_f/\Delta P_0 = f(\varphi)$ when $\varphi > 0.65$, we obtain a formula of the type

$$\Delta P_f/\Delta P_0 = A/(1-\varphi)^n,$$

where n is close to the theoretical exponent obtained in analytical solution of the problem of flow of a mixture with a liquid film.

With values of Re' = 3000–4000, the relation has the form (Fig. 4)

$$\Delta P_f/\Delta P_0 = 1 + 0.61\ W_0''/W_0'. \tag{8}$$

Dependence of the ratio $\Delta P_f/\Delta P_0$ on pressure in this region was not observed.

NOTATION

l—length of stabilizing section; d—tube diameter; ΔP—total pressure drop over experimental section; γ_m—specific weight of mixture; ΔP_f, ΔP_{acc}, ΔP_w—losses in friction, acceleration, and due to weight of mixture; φ_1, φ_2, φ_m—true gas content at beginning and end of experimental section, and mean over experimental section; W_0', W_0'', W_m—reduced velocity of liquid and gas, and mass flow velocity of mixture; g—acceleration due to gravity; $\mathrm{Re}' = W_0' d/\nu'$—Reynolds number of liquid component; $\mathrm{Fr} = (W_0')^2/gd$, $\mathrm{Fr_m} = W_m^2/gd$—Froude number of liquid component and of mixture; ξ_0'—conventional ("reduced" to W_0') resistance coefficient of two-phase flow; ΔP_0—friction losses for motion in a tube of a mass flow of liquid equal to that of the flowing mixture; ν'—kinematic viscosity of liquid component; P—absolute pressure; λ—friction coefficient for motion of a single-phase stream; σ'—surface tension of liquid component; DS—dry substance.

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Technological Institute of the Food Industry, Kiev